Climate Adaptation and War: How Investment in Disaster Preparedness Can Make Conflict More (or Less) Likely

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Abstract

Scholars and practitioners have paid close attention to the effects of climate change on armed conflict. Despite the growing literature, we have a limited understanding of how climate adaptation efforts can affect the risk of conflict. This paper conceptualizes climate adaptation as an investment of resources to alter the distribution of random damages caused by future climate disasters. I present a two-player model in which (i) a climate disaster harms players' military capabilities, and (ii) uneven disaster damages lead to opportunistic aggression due to the military advantage of the side with smaller damage. I consider two cases, one in which only one player can invest and the investor's improved disaster preparedness implies the non-investor's deteriorated preparedness, and the other in which both players can invest and each player's investment only affects her own preparedness. I then show that the security implications of adaptation depend on how it alters the players' vulnerability to future disasters and on how each player's vulnerabilities are correlated with each other. In particular, investment in preparedness can raise the equilibrium probability of war in the first case, even when the same investment technology has a pacifying effect in the second case.

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1 Introduction

In July 2019, Ethiopia deployed the Spyder-MR air defense system near the Grand Ethiopian Renaissance Dam (GERD). Egypt, a downstream country along the Nile, has long expressed concerns about the construction of GERD due to its anticipation that Ethiopia can be tempted to maintain electricity provision by limiting the Nile's flow during a prolonged drought. Not surprisingly, tensions between Ethiopia and Egypt regarding the hydroelectric dam "continue to escalate".[1] Ethiopia's installation of the air defense system suggests that its ability to unilaterally raise its resilience to future droughts leads to a non-negligible military risk, via the anticipation of Egypt incurring a disproportionately high cost of drought. Given that investment in disaster preparedness has security implications, how do such climate adaptation efforts affect the risk of armed conflict between rival political groups?

Although scholars and practitioners have paid close attention to the impact of climate events on the risk of conflict, with some notable exceptions, we have relatively little understanding of how investment in preparedness for these events affects the prospect of war and peace. However, this issue deserves both empirical and theoretical scrutiny. For example, a standard explanation for a positive effect of climate shocks on the risk of conflict focuses on their economic impacts (e.g., Chassang and Padró i Miquel, 2009; Homer-Dixon, 1994). If climate shocks lead to negative economic shocks and negative economic shocks raise the risk of violence, then improving preparedness for climate shocks should decrease the risk of conflict. In fact, a recent paper by Gehring and Schaudt (2026) finds evidence that a preemptive program to mitigate the negative economic effects of drought reduces conflict risks. Nevertheless, the literature on the political economy of climate adaptation is still nascent (Colgan and Genovese, 2025; Grossman et al., 2026; Javeline, 2014). That is, we lack a systematic understanding of how adaptation efforts shape the prospect of war and peace.

This paper theoretically explores how players' ability to invest in their future robustness

[1] https://www.bbc.com/news/articles/cp3dgx36gn5o

to climate shocks affects the risk of military conflict. To this end, I present a simple model in which two political rival groups/countries interact under the shadow of a climate disaster. Building on Sawada (2026), a climate disaster is a random negative shock to each player's military capability. A disaster is random in two senses: both its occurrence and its realized damage to the players are randomly drawn. This negative shock induces war when only one of the players has incurred disproportionately severe damage from the disaster because the relatively undamaged side has a temporary military advantage, which provides her with an opportunistic motive for war.

This paper conceptualizes climate adaptation as *investment to alter the distribution of damages from a future disaster*. That is, one or both players can spend part of their resources to reduce the probability that they experience disproportionately severe damage, conditional on a disaster occurring. This investment in disaster preparedness also has security implications because severe damage on one side can lead to an opportunistic attack from the other side. Thus, in the model, players' adaptation efforts have two security effects: Such an effort renders the investor more robust to future disasters and, as a result, (i) one's own opportunistic aggression against the other more attractive (*temptation* effect) and (ii) aggression from the other side less attractive (*deterrence* effect).

In this paper, I focus on two classes of investment for disaster preparedness. In the first, only one of the players has an investment capability, and investment for further disaster preparedness of one player lowers the robustness of the other. One can think of her as an upstream country along a river with the ability to unilaterally limit its flow when there is a drought (by constructing a dam, for example). Even if there is a drought, the upstream country has a better chance of retaining its military mobilization capability. The other player, a downstream country, might incur more serious damage to its military capabilities after a future drought as a result of the former's investment. Thus, the upstream state can have an opportunistic motive for military aggression if an extreme climate event occurs and it is militarily advantaged due to uneven disaster costs. The downstream state, on the other hand, can have a preventive motive to attack the upstream country before its investment project is completed. Anticipating this preventive motive of the downstream country, the upstream country can be deterred from achieving its first-best investment level.

I show that the implication of such a one-sided investment for the risk of war depends on the specific form of that investment. Namely, if the positive effect of the investment on the upstream country is greater than the negative effect on the downstream country, the overall probability of conflict rises. This is because the above condition is equivalent to the case where the "dam" distorts the distribution of future drought risks in a way that symmetric damages become less likely. Such investment capability decreases players' aggregate welfare whenever it raises the chance of war, compared to a benchmark where neither player can invest.

In the second case, both players have investment capability, and unlike the first case, one player's investment affects only their own robustness to future disasters. An interpretation is that two countries individually construct seawalls on the coastline to prepare for future risks of storms and tsunamis. In this case, under natural conditions, their adaptation efforts reduce the equilibrium probability of war compared to the no-investment benchmark. This stabilizing result arises when the deterrence effect of climate adaptation is greater than the temptation effect.

I also show that unilateral investment in the "dam" case can be destabilizing even when investment in the "seawalls" case has a pacifying effect. These contrasting effects result from the interdependent vulnerability in the first, "dam" case, where the upstream country's improved robustness due to its dam project *must* imply the downstream country's greater vulnerability. In the "seawalls" case, on the other hand, this is not true because the two rival countries may construct seawalls and improve their disaster preparedness simultaneously. Thus, the security implications of climate adaptation efforts depend on both (i) the specific technology of investment and (ii) how the vulnerability of both players is correlated with each other.

1.1 Related Literature

This paper contributes to the literature on the security implications of climate change, natural disasters, and economic shocks (Bas and McLean, 2021; Chassang and Padró i Miquel, 2009; Jun and Sethi, 2021; Kikuta, 2019; Sawada, 2026). This strand of work assumes that political groups are rational, exogenous shocks destroy their material resources, and that they perceive the outcomes of destruction. The closest models are by Bas and McLean (2021) and Sawada (2026), in which a natural disaster damages players' military capabilities. These damages are determined completely exogenously, and players in their models do not have the option to alter how these shocks are materialized. However, if relevant players are rational in the sense that they take into account the risks of such devastating shocks, then it is natural for these players to prepare for these shocks. Thus, the present model departs from existing studies by allowing them to invest their current resources to mitigate future negative shocks.

More generally, the model in this paper also contributes to the political economy literature on armed conflict. Its methodological innovation is to draw new insights on conflict by integrating two families of models. One class of formal models of conflict examines how rapid shifts in military capabilities affect the risk of war (Little and Paine, 2023; Powell, 2004, 2006). The second one allows the players to invest in their future military or agenda-setting power before they decide whether to go to war (Fearon, 2018; Powell, 1993; Ruggiero, 2025; Schram, 2021). If endogenous shifts in power resulting from players' investment are observable by the other side, the investor should refrain from causing a large power shift due to the preventive motive of the declining power, implying that the equilibrium probability of war remains unchanged. Thus, a standard explanation for war caused by endogenous power shifts focuses on strategic uncertainty, i.e., the inability to observe the investment decision (e.g., Debs and Monteiro, 2014; Meirowitz and Sartori, 2008). In this paper, on the other hand, investment is observable but can lead to a higher risk of war because the realization of a rapid power shift (climate shock) is exogenous.

2 The Model

Two players, 1 and 2, interact for two periods ($t \in \{1, 2\}$). In $t = 1$, which I call the "investment phase," players make a costly adaptation effort $e_i \geq 0$. Then, after observing $e = (e_1, e_2)$, each player decides whether to attack the other player or not. The decision is denoted by $a_{i,1} \in \{0, 1\}$, where $a_{i,1} = 1$ denotes "attack" and 0 means "don't attack." If at least one player fights, war erupts, and strategic interactions between players end. If war occurs at $t = 1$, the adaptation project is interrupted and will not have the intended effect specified below. On the other hand, if neither player fights, the game proceeds to $t = 2$, which is called the "disaster phase." At the beginning of the period, Nature draws two types of random variables. The first one is $D \sim \text{Bern}(\pi)$ (as in "disaster"), where $D = 1$ denotes a climate disaster that occurs with probability $\pi \in (0, 1)$. If $D = 1$, Nature draws a pair of random variables, $Z = (Z_1, Z_2)$, where $Z_1, Z_2 \in (0, 1)$, and $Z_i \sim F_i$ with density f_i. Each F_i has full support over $[0, 1]$. Denote the joint distribution by F. They determine the damage to players' military capabilities from the disaster. After observing Z, players choose $a_{i,2} \in \{0, 1\}$. As in the investment phase, the values of one and zero indicate attacking and not attacking the other player, respectively. Then, the game ends, and payoffs are materialized. Figure 1 summarizes the timing of the game.

In this model, climate adaptation efforts are costly investments. Player i pays $\kappa_i e_i$ to achieve adaptation level e_i, where $\kappa_i > 0$. By making an adaptation effort, each player alters the distribution of Z_i. That is, a distribution of Z_i after a higher level of adaptation is strictly first-order stochastic dominant (FOSD) over a distribution after a lower effort.

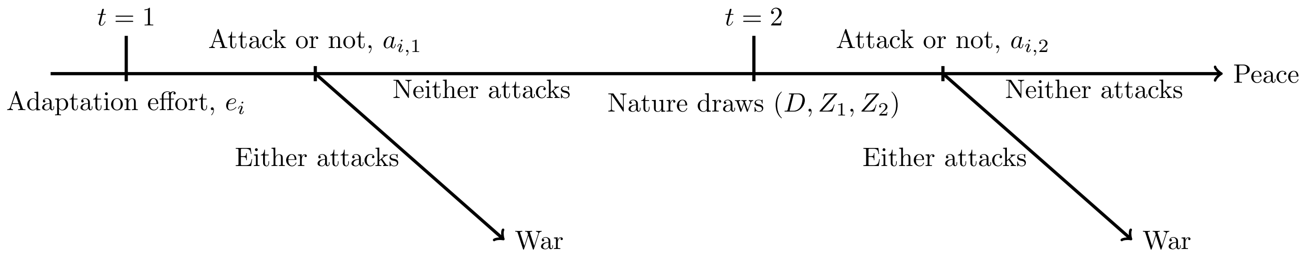

Figure 1: Timeline

Assumption 1. *For any pair of adaptation levels $\overline{e}$ and $\underline{e}$ such that $\overline{e} > \underline{e}$, we have $F_i(z|\overline{e}, e_j) < F_i(z|\underline{e}, e_j)$ for any $z \in (0,1)$. Moreover, $F_i(z|e_i, e_j)$ is continuous in e_i and e_j.*

Assumption 2. $\frac{\partial F_i(z|e_i,e_j)}{\partial e_i} < 0$ *and* $\frac{\partial^2 F_i(z|e_i,e_j)}{\partial e_i^2} > 0$ *for any* $z \in (0,1)$.

War is a costly lottery, and a climate disaster discounts players' military capabilities. Denote by $v_{i,t} \in [0,1]$ the amount of resources that player i possesses in period t. We assume $v_{1,1}, v_{2,1} = \overline{v}$, where $\overline{v} > 0$. On the other hand, the amount of resources in the disaster phase, $v_{i,2}$, depends on D and Z. Specifically, define

$$v_{i,2} = \underbrace{D \times Z_i \times \overline{v}}_{\text{Disaster}} + \underbrace{(1-D) \times \overline{v}}_{\text{No disaster}}.$$

We assume that these resources are convertible to military capability. Letting $p_{i,t}(v_t)$ indicate player i's probability of victory in war at period t, where $v_t = (v_{1,t}, v_{2,t})$, we define

$$p_{i,t}(v_t) = \frac{v_{i,t}}{v_{i,t} + v_{j,t}}.$$

Players' payoffs are as follows. When the game ends peacefully, each player consumes $v_{i,2}$ and both obtain long-term resources $\overline{V} > 0$. When war occurs in period t, the winner obtains an immediate prize $v_{1,2} + v_{2,2}$ and long-term benefits $2 \times \overline{V}$, and the loser receives a payoff of zero. The long-term prize of victory $2\overline{V}$ can be either a material gain, in which the victor obtains the loser's resources, or a long-term policy payoff, in which the victor achieves its ideal policy on a formerly disputed issue. Note that, regardless of the timing of war, the immediate prize is the sum of players' resources at $t = 2$.[2] Thus, each player's resources at $t = 1$ affect the outcome only through $p_{i,1}(v_1)$. Finally, the prize of victory in war, $v_{1,2} + v_{2,2} + 2\overline{V}$, is discounted by $(1 - c)$, where $c \in (0, 1)$ denotes the costliness of war. Thus, player i's expected payoff is

$$\begin{aligned} \text{Peace}: &\quad v_{i,2} + \overline{V} - \kappa_i e_i \\ \text{War at } t: &\quad p_{i,t}(v_t)(1-c)\left(v_{1,2} + v_{2,2} + 2\overline{V}\right) + (1 - p_{i,t}(v_t)) \times 0 - \kappa_i e_i. \end{aligned}$$

The equilibrium concept is subgame perfect equilibrium. To exclude uninteresting cases, we introduce the following tie-breaking rule and an assumption to guarantee that war can occur in equilibrium.

Assumption 3. *When player i is indifferent between fighting and not fighting in $t \in \{1, 2\}$, we assign $a_{i,t} = 0$.*

Assumption 4. *The cost of conflict is sufficiently small: $c < \overline{V} / \left(2\overline{V} + \overline{v}\right)$.*

Next, for simplicity, suppose that players have the identical vulnerability when they do not invest resources. Sawada (2026) examines how exogenously given asymmetric vulnerability affects conflict.

Assumption 5. *If $e_1 = e_2 = 0$, $F_1 = F_2$ and Z_i is symmetric around $1/2$.*

We have assumed that a war interrupts investment. Thus, if there is a war in the investment phase, the expected remaining amount of resources of each player is determined based

[2] If we set $v_{1,1} + v_{2,1}$ as the prize in a war at $t = 1$, we would ignore the effect of a climate disaster. Thus, $v_{1,2} + v_{2,2}$ reflects a natural assumption that a climate disaster can occur even after the political interactions between players ended.

on $e_i = e_j = 0$. For later use, denote the expected payoff from a war in $t = 1$ (excluding sunk investment costs) by $\mathcal{W}^0$:

$$\mathcal{W}^0 = \frac{1}{2} \times (1-c) \times \left(((1-\pi) \times 2 + \pi \times 1)\,\overline{v} + 2\overline{V}\right) = (1-c)\left(\frac{2-\pi}{2}\overline{v} + \overline{V}\right).$$

Note that this war payoff is common across players by Assumption 5. Also, denote player i's expected payoff in the disaster phase (excluding the investment cost) by $\mathcal{V}_i(e)$. Another simplifying assumption is that, given one's own effort e_i, player i becomes willing to fight in $t = 1$ if the other player's effort e_j is sufficiently large:

Assumption 6. *Fixing player i's investment, a large amount of investment by player j renders i's continuation payoff sufficiently small:* $\lim_{e_j \to \infty} \mathcal{V}_i(e_i, e_j) < \mathcal{W}^0$.

3 Analysis

We consider two cases. In the first, only one player can invest in disaster preparedness. In the second case, both players have investment capabilities.

3.1 Dam: One-sided Investment under Interdependent Vulnerability

First, suppose that only player 1 can invest. Player 2 does not have a corresponding decision node to choose an adaptation effort. One interpretation of this special case is that an upstream country along a transboundary river constructs a dam. Once completed, the dam project may render a downstream country more vulnerable to future droughts. Although the downstream country does not have the ability to invest its resources, it can resort to a preventive war before the construction is completed.

In the model, assume the disaster damages are interdependent.

Assumption 7. $Z_2 = 1 - Z_1$.

The assumption implies that a disaster discounts the total amount of resources in the society by half. Then, Z_1 governs the relative damage each player incurs in the sense that player 1 experiencing severe damage implies that player 2's resources are relatively intact, and vice versa. By shifting F_1, player 1 renders player 2 more vulnerable ex ante. For simplicity, we write Z_1 as Z and F_1 as F, respectively. Denote this game by $\Gamma^{\mathbb{1}}$.

Proposition 1 (One-sided investment). *In $\Gamma^{\mathbb{1}}$, the following strategies constitute a subgame perfect equilibrium.*

- *In $t = 1$, player 1 chooses $e^* \equiv \min\{\widehat{e}, \overline{e}\}$, where $\overline{e}$ is the solution to $\mathcal{V}_2(e_1 = \overline{e}) = \mathcal{W}^0$ and $\widehat{e} = \arg\max_{e_1} [\mathcal{V}_1(e_1) - \kappa_1 e_1]$. Then, we have $a_{1,1} = 0$ and $a_{2,1} = \mathbb{1}\{e_1 > \overline{e}\}$.*

- *In $t = 2$, we have $a_{1,2} = \mathbb{1}\{Z > \overline{z}\} D$ and $a_{2,2} = \mathbb{1}\{Z < \underline{z}\} D$, where*

$$\begin{aligned} \overline{z} &= \frac{\overline{V}}{(1-c)\left(\overline{v} + 2\overline{V}\right) - \overline{v}} \quad \textit{and} \\ \underline{z} &= 1 - \overline{z}. \end{aligned}$$

We are interested in how adaptation efforts affect the risk of war. Denote the probability of war with investment level e_1 in $\Gamma^{\mathbb{1}}$ by $\mathcal{P}^{\mathbb{1}}(e_1)$. A useful reference point is the case of no investment. That is, a hypothetical case of $e_1 = 0$ indicates the scenario in which no player has the ability to invest in their preparedness for future disasters. Thus, we are interested in the comparison between $\mathcal{P}^{\mathbb{1}}(0)$ and $\mathcal{P}^{\mathbb{1}}(e^*)$. The following result shows that the effect of adaptation on the risk of war depends on the specific form of investment.

Proposition 2 (Probability of war). *Letting $\Delta(e_1) \equiv F(\overline{z}|e_1) - F(\underline{z}|e_1)$, the equilibrium probability of war in $\Gamma^{\mathbb{1}}$ is greater than the equilibrium probability of war with no investment if and only if $\Delta(e^*) < \Delta(0)$.*

The intuition is as follows. The interval $[\underline{z}, \overline{z}]$ is the range of relative disaster damage within which neither player has the military advantage necessary for opportunistic aggression. Then, if player 1's adaptation effort alters F so that $\Delta(e^*) < \Delta(0)$, the chance of intermediate disaster damage decreases. We already know that Assumption 1 guarantees that, in equilibrium, player 1 and player 2 become more robust and vulnerable, respectively, compared to the benchmark case of $F(z|e_1 = 0)$. The condition $\Delta(e^*) < \Delta(0)$ implies further: if player 1's investment distorts F to the point where an intermediate disaster scenario in which both players incur similar damages is less likely, then war becomes more likely.

Another interpretation of Proposition 2 is that player 1's adaptation is war-inducing when the increase in the likelihood that player 1 obtains a sufficient military advantage outweighs the decrease in the likelihood that player 2 obtains such an advantage. This notion arises by rearranging $\Delta(e^*) < \Delta(0)$. The inequality is equivalent to

$$\underbrace{F(\overline{z}|0) - F(\overline{z}|e^*)}_{\text{Increase in player 1's attack probability}} > \underbrace{F(\underline{z}|0) - F(\underline{z}|e^*)}_{\text{Decrease in player 2's attack probability}},$$

where both sides are positive by Assumption 1. The left-hand (right-hand) side is the increase (decrease) in the probability that player 1 (2) attacks the other player relative to the no-investment benchmark, conditional on $D = 1$. Thus, adaptation has two opposing effects on the risk of war: it makes opportunistic aggression by player 1 more likely while making opportunistic aggression by player 2 less likely. Adaptation increases the equilibrium probability of war if and only if the former effect outweighs the latter.

We are also interested in the welfare effect of adaptation efforts. Because war never erupts in the investment phase ($t = 1$) in equilibrium, we can focus on $\mathcal{V}_1(e_1) - \kappa_1 e_1$ for player 1 and $\mathcal{V}_2(e_1)$ for player 2. Because player 1 can always choose $e_1 = 0$, the effect of investment on player 1's payoff must be nonnegative. By the investment cost $-\kappa_1 e_1$, it is straightforward to see that player 1's payoff other than the cost, $\mathcal{V}_1(e_1)$, is increasing in e_1. On the other hand, because the one-sided investment we are considering here renders player 2 more vulnerable, $\mathcal{V}_2(e_1)$ is decreasing in e_1. The following result presents the effect of one-sided investment on the aggregate welfare of the society.

Proposition 3 (Aggregate welfare)**.** *The condition under which the ex ante aggregate welfare in $\Gamma^{\mathbb{1}}$ becomes strictly smaller than the aggregate welfare in the benchmark without investment is implicitly characterized as*

$$\underbrace{\pi}_{\Pr(D=1)} \times \underbrace{\frac{\Delta\left(e^*\right) - \Delta(0)}{\kappa_1 e^*}}_{\textit{Change in prob. of peace per investment cost}} \times \underbrace{c\left(\overline{v} + 2\overline{V}\right)}_{\textit{Resources destroyed by war}} < 1. \tag{1}$$

Clearly, war-inducing investment is a sufficient condition for a negative welfare effect. Namely, player 1's ability to invest one-sidedly harms society whenever the investment makes war more likely compared to the hypothetical case of no investment ($\Delta\left(e^*\right) - \Delta(0) < 0$). Even when investment reduces the risk of war ($\Delta\left(e^*\right) - \Delta(0) > 0$), players' aggregate welfare decreases if inequality (1) holds, although interpreting it requires caution because the equilibrium investment e^* is a function rather than an exogenous parameter.

Next, we are also interested in how climate risks, π in the model, affect the optimal investment level e^*. This is important because, combined with Proposition 2, this question informs us how the likelihood of climate shocks alters the risk of war through optimal one-sided investment. Namely, if investment is war-inducing ($\Delta\left(e^*\right) < \Delta(0)$) and the equilibrium investment (e^*) is increasing in the probability of climate shocks (π), then a greater risk of climate shocks is also war-inducing. On the other hand, if investment is war-inducing but e^* is decreasing in π, $\mathcal{P}^{\mathbb{1}}$ could be decreasing in π. Thus, although π must decrease the aggregate welfare in society, a higher risk of climate disasters might be peace-promoting in equilibrium. The following result shows that the effect of π on the optimal investment level e^* can be nonmonotonic.

Proposition 4 (Nonmonotonic effect of disaster risks and optimal investment)**.** *If* $\mathcal{V}_2\left(\widehat{e}\right)\big|_{\pi=1} < \frac{1}{2}(1-c)\left(\overline{v}+2\overline{V}\right)$*, there exists* $\widehat{\pi} \in (0,1)$ *such that* e^* *is increasing in* π *for* $\pi \leq \widehat{\pi}$ *and decreasing in* π *for* $\pi \geq \widehat{\pi}$*. Otherwise,* e^* *is increasing in* π*.*

This nonmonotonicity arises because climate adaptation in this model has two opposing effects. First, because player 1's investment deters player 2's potential aggression in the presence of a climate shock (deterrence effect), an increase in the risk of a shock (π) results in an increase in the equilibrium investment. Second, player 1's greater effort also means a greater temptation to attack player 2 when there is a disaster (temptation effect). When a climate shock is likely, i.e., when π is large, the preventive motive of player 2 deters player 1 from achieving the first-best investment level, $\widehat{e}$. Consequently, the equilibrium investment e^* increases in π when it is small, and decreases in π for $\pi \geq \widehat{\pi}$.

3.2 Seawalls: Two-sided Investment under Independent Vulnerability

Now we assume that both players can invest in their preparedness for future climate shocks. We also suppose that each player's damage from a climate shock is independent. One interpretation is that two neighboring countries are located along a coastal region susceptible to storms and tsunamis. Because the damage from these climate disasters to each country depends on the specific weather event, more severe damage to one country does not necessarily imply severe damage to the other. If they build seawalls along the coast in their territory, such investment mitigates future disaster damage of their own but not the other state's.

In the model, we assume the following. First, denoting $e = (e_1, e_2)$, the damage from a disaster to each player is independently drawn.

Assumption 8. $F(z_1, z_2|e) = F_1(z_1|e)F_2(z_2|e)$.

Second, player i's investment e_i affects only its own future damage, F_i, but not the other's.

Assumption 9. $F_i(z_i|e_i, e_j) = F_i(z_i|e_i)$.

We label this case as Γ^2. In a similar manner as the one-sided investment case, let $\mathcal{V}_i^2(e_i, e_j)$ be player i's continuation payoff (excluding investment costs). Also, denote three key thresholds in players' investment levels. First, let $\widehat{e}_i(e_j) = \arg\max_{e_i} [\mathcal{V}_i^2(e_i, e_j) - \kappa_i e_i]$ be the optimal level of e_i, unconstrained by the possibility of war in $t = 1$. Second, $\overline{e}_i(e_j)$ is the solution to $\mathcal{V}_j^2\left(e_i = \overline{e}_i(e_j), e_j\right) = \mathcal{W}^0$, which is i's investment level at which player j is indifferent between fighting and not fighting in $t = 1$. Third, $\underline{e}_i(e_j)$ is the solution to $\mathcal{V}_i^2\left(e_i = \underline{e}_i(e_j), e_j\right) = \mathcal{W}^0$, which is i's investment level at which i herself is indifferent between fighting and not fighting in $t = 1$. These are the candidates for the equilibrium investment.

Proposition 5 (Two-sided independent investment)**.** *In Γ^2, the following strategies constitute a subgame perfect equilibrium.*

- *In $t = 1$, player i chooses investment level e_i^2, where (e_1^2, e_2^2) solves*

$$\begin{cases} e_1^2 = \max\left\{\underline{e}_1\left(e_2^2\right), \min\left\{\widehat{e}_1\left(e_2^2\right), \overline{e}_1\left(e_2^2\right)\right\}\right\} \\ e_2^2 = \max\left\{\underline{e}_2\left(e_1^2\right), \min\left\{\widehat{e}_2\left(e_1^2\right), \overline{e}_2\left(e_1^2\right)\right\}\right\}. \end{cases}$$

  Then, we have $a_{i,1} = \mathbb{1}\left\{e_j > \overline{e}_j(e_i)\right\}$.

- *In $t = 2$, player i chooses $a_{i,2} = \mathbb{1}\left\{z_j < \underline{z}_j(z_i)\right\} D$, where*

$$\underline{z}_j(z_i) = \frac{(1 - 2c)\overline{V} - c\overline{v}z_i}{\overline{V} + c\overline{v}z_i}\, z_i.$$

As in the previous case, in equilibrium, war occurs with positive probability only when $D = 1$ at $t = 2$. The condition under which player i fights in $t = 1$, $e_j > \overline{e}_j(e_i)$, never holds in equilibrium. Unlike the previous case, however, note that the threshold in the other player's remaining resources below which player i is willing to attack when $D = 1$, $\underline{z}_j(z_i)$, depends on i's own resources. By $\frac{(1-2c)\overline{V}-c\overline{v}z_i}{\overline{V}+c\overline{v}z_i} < 1$, it is easy to see $\underline{z}_j(z_i) < z_i$. Hence, at most one player (the one with disproportionately small damage) has the incentive to attack the other in equilibrium.

As in the case of one-sided investment, we are interested in the equilibrium probability of war. To consider this, we denote by $\rho_i(e_1, e_2)$ the probability that player i attacks player j in $t = 2$ conditional on $D = 1$:

$$\begin{aligned}
\rho_i(e_1, e_2) &= \Pr(a_{i,2} = 1 | D = 1, e_1, e_2) \\
&= \Pr\left(Z_j < \underline{z}_j(z_i) | e_1, e_2\right) \\
&= \mathbb{E}_{Z_i \sim F_i(\cdot|e_i)}\left[F_j\left(\underline{z}_j(z_i) \middle| e_j\right)\right] \\
&= \int_0^1 F_j\left(\frac{z_i\left((1-2c)\overline{V} - c\overline{v}z_i\right)}{\overline{V} + c\overline{v}z_i}\middle| e_j\right) f_i(z_i|e_i)\mathrm{d}z_i.
\end{aligned}$$

Then, with the no-investment benchmark where $e_i = e_j = 0$, we define the temptation effect, denoted by $\tau\left(e^2\right)$, and the deterrence effect, denoted by $\delta\left(e^2\right)$, of climate adaptation:

$$\begin{aligned}
\tau\left(e^2\right) &= \underbrace{\rho_1\left(e_1^2, 0\right) - \rho_1\left(0, 0\right)}_{\text{Player 1's temptation}} + \underbrace{\rho_2\left(0, e_2^2\right) - \rho_2\left(0, 0\right)}_{\text{Player 2's temptation}} \\
\delta\left(e^2\right) &= \underbrace{\rho_1\left(e_1^2, 0\right) - \rho_1\left(e_1^2, e_2^2\right)}_{\text{Player 2's deterrence against player 1}} + \underbrace{\rho_2\left(0, e_2^2\right) - \rho_2\left(e_1^2, e_2^2\right)}_{\text{Player 1's deterrence against player 2}}.
\end{aligned}$$

Proposition 6 (Probability of war)**.** *The equilibrium probability of war in Γ^2 is greater than the probability of war in the no-investment benchmark if and only if $\tau\left(e^2\right) > \delta\left(e^2\right)$.*

This result is intuitive: The equilibrium probability of war is greater than that of the hypothetical no-investment case if and only if the temptation effect of climate adaptation is greater than the deterrence effect. To see this, consider the equilibrium probability of player i's aggression, $\rho_i\left(e_1^2, e_2^2\right)$, and two hypothetical benchmarks, $\rho_i(0, 0)$ and $\rho_i\left(e_i^2, 0\right)$. Beginning from the no-investment benchmark with $e = (0, 0)$, suppose player i moves from $e_i = 0$ to the equilibrium investment level $e_i = e_i^2$. In this case, the probability of player i's aggression (conditional on $D = 1$) increases from $\rho_i(0,0)$ to $\rho_i(e_i^2, 0)$. The rise in the probability indicates the temptation effect of i's investment. Next, suppose player j now achieves her equilibrium investment, $e_j = e_j^2$. Then, player i's aggression probability decreases from $\rho_i(e_i^2, 0)$ to $\rho_i(e_1^2, e_2^2)$. The decline in i's attack probability represents the deterrence effect of player j's investment. In sum, player i becomes more aggressive than in the hypothetical no-investment case when the rise in the attack probability (i's temptation) is greater than the decline (j's deterrence). Therefore, the equilibrium probability of war is greater than in the no-investment benchmark if and only if the sum of the players' temptation effects is greater than the sum of the deterrence effects.

An important result arises from comparing the one-sided and two-sided investment models. Namely, under natural assumptions, the two-sided investment case must have a pacifying effect. The assumptions are as follows. First, f_i satisfies the monotone likelihood ratio property (MLRP): For each $i \in \{1,2\}$ and for any $z'' > z'$ and $e_i'' > e_i'$,

$$\frac{f_i(z''|e'')}{f_i(z''|e')} \geq \frac{f_i(z'|e'')}{f_i(z'|e')}. \tag{2}$$

Second, suppose (i) $e = (0,0)$, (ii) player i's proportion of remaining resources after a disaster, Z_i is z, and (iii) player j has incurred a weakly greater damage, i.e., $Z_j \leq z$. Then, the conditional probability that player i attacks j given $Z_j \leq z$ is weakly decreasing in z:

$$\frac{\partial}{\partial z} \Pr\left(Z_j < \underline{z}_j(z) \middle| Z_j \leq z, e_j = 0\right) = \frac{\partial}{\partial z} \frac{F_j\left(\underline{z}_j(z)|0\right)}{F_j(z|0)} \leq 0. \tag{3}$$

Proposition 7 (Pacifying two-sided investment). *If F_i satisfies MLRP (2) and nonincreasing conditional attack probability (3) for each $i \in \{1,2\}$, then the equilibrium probability of war in Γ^2 is always weakly smaller than the probability of war in the no-investment benchmark.*

Importantly, there exist cases where climate adaptation is destabilizing in the one-sided

	No investment benchmark	Equilibrium	Change in war probability
Dam	0.165	**0.178**	+1.3 pp
Seawalls	0.138	**0.064**	−7.4 pp

Table 1: Numerical example of opposing security implications of adaptation

investment model $\Gamma^{\mathbb{1}}$ even when F_1 satisfies the conditions.

Example (Destabilizing one-sided investment despite pacifying two-sided investment). Let $F_i(z|e_i) = z^{1+2e_i}$, $\overline{v} = 1$, $\overline{V} = 1$, $c = 0.1$, $\pi = 0.2$, and $\kappa_i = 0.05$ for each $i \in \{1, 2\}$, where $F_i(z|e_i) = z^{1+2e_i}$ satisfies both MLRP (2) and nonincreasing conditional attack probability (3). Under this F_i and the above parameters, consider the one-sided investment case. The ex ante probability of war for the hypothetical no-investment benchmark is approximately 0.165. The equilibrium probability with $e_1 = e^*$ is 0.178, where player 1's investment has a negative security implication. On the other hand, in the two-sided investment case, the no-investment benchmark yields the ex ante probability of war of 0.138. The equilibrium probability under $e = (e_1^{\mathbb{2}}, e_2^{\mathbb{2}})$ is approximately 0.064, where climate adaptation is pacifying. In this example, note that the temptation and deterrence effects of adaptation are

$$\tau\left(e^{\mathbb{2}}\right) \approx 0.293 < \delta\left(e^{\mathbb{2}}\right) \approx 0.664.$$

Proposition 7 and the above example suggest that the security implications of climate adaptation depend not only on the specific technology of adaptation, i.e., $F_i(\cdot|e_i)$, but also on what player i's vulnerability means for player j's vulnerability. That is, the destabilizing effect of the one-sided investment case, despite the pacifying effect of the two-sided investment case, arises from the interdependent vulnerability in the former, where the improved robustness of an upstream country along a dam means the deteriorated disaster preparedness of a downstream country.

To see this, note that Proposition 7 does not depend on the cost of adaptation, κ_i. This implies that, as long as (2) and (3) hold and Z_1 and Z_2 are independent, the result remains true even when the adaptation cost of one of the players is prohibitively high, and thus, the model is essentially a one-sided investment case. Rather, the contrasting security implications are due to the fact that, in the one-sided investment case where $Z_2 = 1 - Z_1$, a more robust player i must imply a more vulnerable j. In the two-sided investment case, this is not necessarily true. Namely, when player i becomes more robust to future disaster risks, player j may also be more robust. Hence, a greater deterrence effect and a smaller temptation effect can be compatible in the two-sided investment case.

4 Conclusion

This paper has considered the security implications of climate adaptation. In the formal model, investment in disaster preparedness affects the risk of war by altering the distribution of future military capabilities in the aftermath of a climate shock. In particular, this paper presents two cases whose interpretations are (i) a dam project that renders an upstream country more robust to future drought risks at the expense of a downstream country and (ii) the construction of seawalls in two neighboring rival countries, in which the improved disaster preparedness of the coastline in one's territory does not affect that of the other country. The model shows that how climate adaptation affects the prospect of war and peace depends on the specific investment technology and the relationship between the two players' disaster vulnerabilities. That is, even when a certain investment technology has a pacifying effect in the "seawalls" case, the same adaptation method can increase the risk of war in the "dam" case because the upstream country's improved robustness to droughts necessarily harms the drought preparedness of the downstream country.

A natural extension is to consider a case with spillover effects of the adaptation efforts. In the two-sided investment ("seawalls") case, each player's adaptation effort only improves one's own preparedness. However, if a player's investment not only enhances one's own robustness to disasters but also improves the other's, the security implications and welfare effects are not obvious because there can be free-riding. Identifying the conditions under which strategic complementarity or substitutability emerges should lead to important policy implications.

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A Proofs

Proof of Proposition 1. We solve the game by backward induction. Consider $t = 2$. When $D = 0$, each player's resource is $\overline{v}$. If the game ends peacefully, each receives $\overline{v} + \overline{V}$. On the other hand, the expected payoff from war is $\frac{1}{2}\left(2\overline{v} + 2\overline{V}\right)(1-c)$. Thus, neither player fights when $D = 0$. When $D = 1$, player i fights if and only if

$$\underbrace{v_{i,2} + \overline{V}}_{\text{Peace}} < \underbrace{\frac{v_{i,2}}{v_{1,2} + v_{2,2}}\left(v_{1,2} + v_{2,2} + 2\overline{V}\right)(1-c)}_{\text{War}}. \tag{4}$$

Because $v_{i,2} = \mathbb{1}\{i = 1\} z\overline{v} + \mathbb{1}\{i = 2\}(1-z)\overline{v}$ when $(D, Z) = (1, z)$, players 1 and 2 fight if and only if

$$\begin{aligned} z &> \frac{\overline{V}}{(1-c)\left(\overline{v} + 2\overline{V}\right) - \overline{v}} \equiv \overline{z} \text{ and} \\ z &< 1 - \overline{z} \equiv \underline{z}. \end{aligned}$$

Consider $t = 1$. I first show that there exists a unique threshold in e_1 above which player 2 fights in $t = 1$. Player 2 attacks player 1 if and only if

$$\underbrace{p_{2,1}(v_1)\left(\pi \times \overline{v} + (1-\pi) \times 2\overline{v} + 2\overline{V}\right)(1-c)}_{\text{Preventive war}} = (1-c)\left(\frac{2-\pi}{2}\overline{v} + \overline{V}\right) > \underbrace{\mathcal{V}_2(e_1)}_{\text{Continuation}}.$$

We have

$$\mathcal{V}_2(e_1) = (1-\pi)\left(\overline{v}+\overline{V}\right)+\pi\left[\begin{array}{l}\underbrace{\left(F\left(\overline{z}|e_1\right)-F\left(\underline{z}|e_1\right)\right)}_{\text{Peace}}\mathbb{E}\left[(1-Z)\overline{v}+\overline{V}\middle|e_1, Z\in[\underline{z},\overline{z}]\right]\\ +\underbrace{F\left(\underline{z}|e_1\right)}_{\text{2 attacks 1}}\mathbb{E}\left[1-Z|e_1, Z<\underline{z}\right]\left(\overline{v}+2\overline{V}\right)(1-c)\\ +\underbrace{\left(1-F\left(\overline{z}|e_1\right)\right)}_{\text{1 attacks 2}}\mathbb{E}\left[1-Z|e_1, Z>\overline{z}\right]\left(\overline{v}+2\overline{V}\right)(1-c)\end{array}\right]$$

$$= (1-\pi)\left(\overline{v}+\overline{V}\right) +\pi\left[\begin{array}{l}\underbrace{\int_{\underline{z}}^{\overline{z}}\left((1-z)\overline{v}+\overline{V}\right)f(z|e_1)\mathrm{d}z}_{(i)}\\ +\underbrace{\left(\overline{v}+2\overline{V}\right)(1-c)\left(\int_0^{\underline{z}}(1-z)f(z|e_1)\mathrm{d}z+\int_{\overline{z}}^1(1-z)f(z|e_1)\mathrm{d}z\right)}_{(ii)}\end{array}\right]. \quad (5)$$

By integration by parts, terms (i) and (ii), respectively, simplify to

$$\begin{aligned}(i)\qquad & \overline{v}\int_{\underline{z}}^{\overline{z}}(1-z)f(z|e_1)\mathrm{d}z+\overline{V}\left(F\left(\overline{z}|e_1\right)-F\left(\underline{z}|e_1\right)\right)\\ = {} & \overline{v}\left(\left(1-\overline{z}\right)F\left(\overline{z}|e_1\right)-\left(1-\underline{z}\right)F\left(\underline{z}|e_1\right)+\int_{\underline{z}}^{\overline{z}}F(z|e_1)\mathrm{d}z\right)+\overline{V}\left(F\left(\overline{z}|e_1\right)-F\left(\underline{z}|e_1\right)\right)\\ (ii)\qquad & (1-c)\left(\overline{v}+2\overline{V}\right)\left(\left(1-\underline{z}\right)F\left(\underline{z}|e_1\right)+\int_0^{\underline{z}}F(z|e_1)\mathrm{d}z-\left(1-\overline{z}\right)F\left(\overline{z}|e_1\right)+\int_{\overline{z}}^1F(z|e_1)\mathrm{d}z\right).\end{aligned}$$

Then, by $\underline{z}=1-\overline{z}$ and $\overline{z}=\frac{\overline{V}}{(1-c)\left(\overline{v}+2\overline{V}\right)-\overline{v}}$, adding (i) and (ii) yields

$$\begin{aligned}& \overline{v}\int_{\underline{z}}^{\overline{z}}F(z|e_1)\mathrm{d}z+(1-c)\left(\overline{v}+2\overline{V}\right)\left(\int_0^{\underline{z}}F(z|e_1)\mathrm{d}z+\int_{\overline{z}}^1F(z|e_1)\mathrm{d}z\right)\\ & +F\left(\underline{z}|e_1\right)\left[\underbrace{\overbrace{\left(1-\underline{z}\right)}^{=\overline{z}}\left((1-c)\left(\overline{v}+2\overline{V}\right)-\overline{v}\right)-\overline{V}}_{=0}\right]+F\left(\overline{z}|e_1\right)\left[\overline{V}-\left(1-\overline{z}\right)\left((1-c)\left(\overline{v}+2\overline{V}\right)-\overline{v}\right)\right]\\ = {} & \overline{v}\int_{\underline{z}}^{\overline{z}}F(z|e_1)\mathrm{d}z+(1-c)\left(\overline{v}+2\overline{V}\right)\left(\int_0^{\underline{z}}F(z|e_1)\mathrm{d}z+\int_{\overline{z}}^1F(z|e_1)\mathrm{d}z\right)+c\left(\overline{v}+2\overline{V}\right)F\left(\overline{z}|e_1\right).\end{aligned}$$

Thus, we obtain

$$\begin{aligned}\mathcal{V}_2(e_1) &= (1-\pi)\left(\overline{v}+\overline{V}\right) \\ &+\pi\left[\overline{v}\int_{\underline{z}}^{\overline{z}} F(z|e_1)\mathrm{d}z + \left(\overline{v}+2\overline{V}\right)\left(cF\left(\overline{z}|e_1\right)+(1-c)\left(\int_0^{\underline{z}} F(z|e_1)\mathrm{d}z + \int_{\overline{z}}^1 F(z|e_1)\mathrm{d}z\right)\right)\right].\end{aligned} \tag{6}$$

Then, by Assumption 1, $\mathcal{V}_2$ is decreasing in e_1. Also, by Assumption 1 and expression (6), $\mathcal{V}_2$ is continuous in e_1.

Next, player 2 does not fight when $e_1 = 0$. To see this, first observe $\mathcal{V}_1(0) = \mathcal{V}_2(0)$ by Assumptions 5 and 7. Then, the sum of the players' continuation payoffs is

$$\begin{aligned}\mathcal{V}_1(0)+\mathcal{V}_2(0) &= (1-\pi)\times 2\left(\overline{v}+\overline{V}\right) \\ &+\pi\left[\underbrace{\left(F\left(\overline{z}|0\right)-F\left(\underline{z}|0\right)\right)\left(\overline{v}+2\overline{V}\right)}_{\text{Peace}}+\underbrace{\left(F\left(\underline{z}|0\right)+1-F\left(\overline{z}|0\right)\right)(1-c)\left(\overline{v}+2\overline{V}\right)}_{\text{War}}\right].\end{aligned}$$

Because dividing this by 2 yields $\mathcal{V}_2(0)$, we obtain

$$\begin{aligned}\mathcal{V}_2(0) &= (1-\pi)\left(\overline{v}+\overline{V}\right)+\frac{1}{2}\pi\left(\overline{v}+2\overline{V}\right)\left[F\left(\overline{z}|0\right)-F\left(\underline{z}|0\right)+(1-c)\left(F\left(\underline{z}|0\right)+1-F\left(\overline{z}|0\right)\right)\right] \\ &= (1-\pi)\left(\overline{v}+\overline{V}\right)+\frac{1}{2}\pi\left(\overline{v}+2\overline{V}\right)\left[1-c\left(F\left(\underline{z}|0\right)+1-F\left(\overline{z}|0\right)\right)\right] \\ &> (1-\pi)\left(\overline{v}+\overline{V}\right)\times(1-c)+\frac{1}{2}\pi\left(\overline{v}+2\overline{V}\right)\times(1-c) \\ &= \underbrace{(1-c)\left(\frac{2-\pi}{2}\overline{v}+\overline{V}\right)}_{\text{War in } t=1}.\end{aligned} \tag{7}$$

Combined with Assumption 6, we have $\mathcal{V}_2(e_1=0) > (1-c)\left(\frac{2-\pi}{2}\overline{v}+\overline{V}\right) > \mathcal{V}_2(e_1=e^{\ddagger})$ for some sufficiently large $e^{\ddagger}$. Thus, by the intermediate value theorem and by strict FOSD (Assumption 1), there exists a unique threshold $\overline{e}$ such that

$$\mathcal{V}_2\left(\overline{e}\right)=(1-c)\left(\frac{2-\pi}{2}\overline{v}+\overline{V}\right). \tag{8}$$

Consequently, player 2's equilibrium decision in $t = 1$ is $a_{2,1} = \mathbb{1}\{e_1 > \overline{e}\}$.

Consider player 1's decision whether to fight. In a similar manner as $\mathcal{V}_2(e_1)$, we have

$$\begin{aligned}\mathcal{V}_1(e_1) \;\;=\;\; & (1-\pi)\left(\overline{v}+\overline{V}\right)+\pi\left[-\overline{v}\int_{\underline{z}}^{\overline{z}} F(z|e_1)\mathrm{d}z\right. \\ & \left.-\left(\overline{v}+2\overline{V}\right)\left(cF\left(\underline{z}|e_1\right)+(1-c)\left(\int_0^{\underline{z}} F(z|e_1)\mathrm{d}z+\int_{\overline{z}}^1 F(z|e_1)\mathrm{d}z-1\right)\right)\right],\end{aligned} \tag{9}$$

which is increasing in e_1 by Assumption 1. Hence, player 1 does not fight in $t = 1$ because, for any $e_1 \geq 0$,

$$\begin{aligned}\mathcal{V}_1(e_1)-\kappa_1 e_1 \quad & \geq \quad \mathcal{V}_1(0)-\kappa_1 e_1 \\ & \underbrace{=}_{\text{Assumption 5}} \quad \mathcal{V}_2(0)-\kappa_1 e_1 \\ & \underbrace{>}_{\text{Inequality (7)}} \quad \underbrace{(1-c)\left(\frac{2-\pi}{2}\overline{v}+\overline{V}\right)}_{\text{War in } t=1}-\kappa_1 e_1.\end{aligned}$$

Lastly, consider player 1's optimal investment, denoted by e^*. Because

$$\begin{aligned}\mathcal{V}_1''(e_1) \;\;=\;\; & -\pi\left[\overline{v}\int_{\underline{z}}^{\overline{z}} \underbrace{\frac{\partial^2 F\left(z|e_1\right)}{\partial e_1^2}}_{>0}\mathrm{d}z\right. \\ & \left.+\left(\overline{v}+2\overline{V}\right)\left(c\underbrace{\frac{\partial^2 F\left(\underline{z}|e_1\right)}{\partial e_1^2}}_{>0}+(1-c)\left(\int_0^{\underline{z}}\underbrace{\frac{\partial^2 F\left(z|e_1\right)}{\partial e_1^2}}_{>0}\mathrm{d}z+\int_{\overline{z}}^1\underbrace{\frac{\partial^2 F\left(z|e_1\right)}{\partial e_1^2}}_{>0}\mathrm{d}z\right)\right)\right] \\ < \;\; & 0\end{aligned}$$

by Assumption 2, $\mathcal{V}_1(e_1)-\kappa_1 e_1$ is strictly concave. Also, $\mathcal{V}_1(e_1)$ is bounded above, and hence $\mathcal{V}_1(e_1)-\kappa_1 e_1$ diverges to $-\infty$ as e_1 approaches ∞. Thus, there exists a unique optimum $\widehat{e} = \arg\max_{e_1 \geq 0}\left[\mathcal{V}_1(e_1)-\kappa_1 e_1\right]$. Because investing $e_1 > \overline{e}$ and inducing a preventive war is not optimal for player 1, we obtain $e^* = \min\{\widehat{e}, \overline{e}\}$. $\square$

Proof of Proposition 2. By Proposition 1, war erupts on the equilibrium path when $D = 1$ and $Z > \overline{z}$ or $Z < \underline{z}$. Thus, the equilibrium probability of war is

$$\Pr(D = 1)(1 - \Pr(Z \in [\underline{z}, \overline{z}]|e_1)) = \pi(1 - F(\overline{z}|e_1) + F(\underline{z}|e_1)) \equiv \mathcal{P}^{\mathbb{1}}(e_1).$$

Also, observe that the probability of war in the no-investment benchmark is equal to $\mathcal{P}^{\mathbb{1}}(0)$. Thus, the equilibrium probability of war in $\Gamma^{\mathbb{1}}$ is greater than the probability of war in the case without investment if and only if

$$\begin{aligned} \mathcal{P}^{\mathbb{1}}(e^*) &> \mathcal{P}^{\mathbb{1}}(0) \\ \underbrace{F(\overline{z}|0) - F(\underline{z}|0)}_{\Delta(0)} &> \underbrace{F(\overline{z}|e^*) - F(\underline{z}|e^*)}_{\Delta(e^*)}, \end{aligned}$$

which completes the proof. □

Proof of Proposition 3. By Proposition 1, we know that war never occurs in $t = 1$ in equilibrium. Thus, players' equilibrium payoffs are $\mathcal{V}_1(e^*) - \kappa_1 e^*$ for player 1 and $\mathcal{V}_2(e^*)$ for player 2, respectively. Now fix some investment level e_1. The aggregate amount of resources in the society is $2 \times (\overline{v} + \overline{V})$ when $D = 0$, $\overline{v} + 2 \times \overline{V}$ when $D = 1$ and war does not occur, and $(1 - c)(\overline{v} + 2\overline{V})$ when $D = 1$ and war occurs. Thus, the aggregate welfare is

$$\begin{aligned} \mathcal{V}_1(e_1) - \kappa_1 e_1 + \mathcal{V}_2(e_1) &= (1 - \pi) \times 2(\overline{v} + \overline{V}) + \pi\Delta(e_1) \times (\overline{v} + 2\overline{V}) \\ &\quad + \pi(1 - \Delta(e_1)) \times (1 - c)(\overline{v} + 2\overline{V}) - \kappa_1 e_1 \\ &= 2(1 - \pi)(\overline{v} + \overline{V}) + \pi(1 - c(1 - \Delta(e_1)))(\overline{v} + 2\overline{V}) - \kappa_1 e_1. \end{aligned}$$

Thus, the aggregate welfare when $e_1 = e^*$ is strictly smaller than that in the case of $e_1 = 0$

when

$$\begin{aligned}
\mathcal{V}_1(e^*) - \kappa_1 e^* + \mathcal{V}_2(e^*) &< \mathcal{V}_1(0) + \mathcal{V}_2(0) \\
(\Delta(e^*) - \Delta(0))\, \pi c \left(\overline{v} + 2\overline{V}\right) &< \kappa_1 e^*,
\end{aligned}$$

which is equivalent to the condition in the statement. $\square$

Proof of Proposition 4. Consider $\widehat{e}$. Because we consider $\overline{e}$ below, ignore the possibility of preventive war for the moment. Then, we know that there is a unique optimal investment by Proposition 1. At such $e_1 = \widehat{e}$, we have

$$\left.\frac{\partial \mathcal{V}_1(e_1)}{\partial e_1}\right|_{e_1=\widehat{e}} - \kappa_1 = 0. \tag{10}$$

Differentiating (9) with respect to e_1 yields

$$\begin{aligned}
\frac{\partial \mathcal{V}_1(e_1)}{\partial e_1} = & -\pi \left[\overline{v} \int_{\underline{z}}^{\overline{z}} \frac{\partial F(z|e_1)}{\partial e_1} \mathrm{d}z \right. \\
& \left. + \left(\overline{v} + 2\overline{V}\right) \left(c \frac{\partial F(\underline{z}|e_1)}{\partial e_1} + (1-c) \left(\int_0^{\underline{z}} \frac{\partial F(z|e_1)}{\partial e_1} \mathrm{d}z + \int_{\overline{z}}^1 \frac{\partial F(z|e_1)}{\partial e_1} \mathrm{d}z \right) \right) \right].
\end{aligned}$$

Observe that $\frac{\partial \mathcal{V}_1(e_1)}{\partial e_1}$ is linear in π. Hence, we obtain

$$\frac{\partial^2 \mathcal{V}_1(e_1)}{\partial e_1 \partial \pi} = \frac{1}{\pi} \frac{\partial \mathcal{V}_1(e_1)}{\partial e_1}.$$

Because $\left.\frac{\partial^2 \mathcal{V}_1(e_1)}{\partial e_1^2}\right|_{e_1=\widehat{e}} < 0$ by Assumption 2, the implicit function theorem yields

$$\frac{\partial \widehat{e}}{\partial \pi} = -\frac{\left.\frac{\partial^2 \mathcal{V}_1(e_1)}{\partial e_1 \partial \pi}\right|_{e_1=\widehat{e}}}{\left.\frac{\partial^2 \mathcal{V}_1(e_1)}{\partial e_1^2}\right|_{e_1=\widehat{e}}} = -\frac{\kappa_1}{\pi \left.\frac{\partial^2 \mathcal{V}_1(e_1)}{\partial e_1^2}\right|_{e_1=\widehat{e}}}.$$

by (10). This implies $\frac{\partial \widehat{e}}{\partial \pi} > 0$.

Next, consider $\overline{e}$. By (8), we have $\mathcal{V}_2(\overline{e}) = (1-c)\left(\frac{2-\pi}{2}\overline{v} + \overline{V}\right)$. We are interested in $\frac{\partial \overline{e}}{\partial \pi}$. By the chain rule, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}\pi}\mathcal{V}_2(\overline{e}) = -\frac{1-c}{2}\overline{v} = \left.\frac{\partial \mathcal{V}_2(e_1)}{\partial \pi}\right|_{e_1=\overline{e}} + \left.\frac{\partial \mathcal{V}_2(e_1)}{\partial e_1}\right|_{e_1=\overline{e}}\frac{\partial \overline{e}}{\partial \pi}. \tag{11}$$

Differentiating (6) with respect to e_1 leads to

$$\begin{aligned}\frac{\partial \mathcal{V}_2(e_1)}{\partial e_1} &= \pi\left[\overline{v}\int_{\underline{z}}^{\overline{z}}\frac{\partial F(z|e_1)}{\partial e_1}\mathrm{d}z\right. \\ &\quad \left. + \left(\overline{v} + 2\overline{V}\right)\left(c\frac{\partial F(\overline{z}|e_1)}{\partial e_1} + (1-c)\left(\int_0^{\underline{z}}\frac{\partial F(z|e_1)}{\partial e_1}\mathrm{d}z + \int_{\overline{z}}^{1}\frac{\partial F(z|e_1)}{\partial e_1}\mathrm{d}z\right)\right)\right] \\ &< 0\end{aligned}$$

by Assumption 2. Next, by differentiating (6) with respect to π, we obtain

$$\begin{aligned}\frac{\partial \mathcal{V}_2(e_1)}{\partial \pi} &= -\left(\overline{v} + \overline{V}\right) + \overline{v}\int_{\underline{z}}^{\overline{z}} F(z|e_1)\mathrm{d}z \\ &\quad + \left(\overline{v} + 2\overline{V}\right)\left(cF(\overline{z}|e_1) + (1-c)\left(\int_0^{\underline{z}} F(z|e_1)\mathrm{d}z + \int_{\overline{z}}^{1} F(z|e_1)\mathrm{d}z\right)\right).\end{aligned}$$

By (6), it is straightforward to see

$$\frac{\partial \mathcal{V}_2(e_1)}{\partial \pi} = \frac{1}{\pi}\left(\mathcal{V}_2(e_1) - \left(\overline{v} + \overline{V}\right)\right).$$

Finally, observe

$$\begin{aligned}\left.\frac{\partial \mathcal{V}_2(e_1)}{\partial \pi}\right|_{e_1=\overline{e}} &= \frac{1}{\pi}\left(\mathcal{V}_2(\overline{e}) - \left(\overline{v} + \overline{V}\right)\right) \\ &= \frac{1}{\pi}\left[(1-c)\left(\frac{2-\pi}{2}\overline{v} + \overline{V}\right) - \left(\overline{v} + \overline{V}\right)\right] \\ &= -\frac{c\left(\overline{v} + \overline{V}\right)}{\pi} - \frac{1-c}{2}\overline{v}.\end{aligned}$$

Substituting this into (11), we obtain

$$\begin{aligned}
-\frac{c\left(\overline{v}+\overline{V}\right)}{\pi}-\frac{1-c}{2}\overline{v}+\left.\frac{\partial\mathcal{V}_2(e_1)}{\partial e_1}\right|_{e_1=\overline{e}}\frac{\partial\overline{e}}{\partial\pi} &= -\frac{1-c}{2}\overline{v}\\
\frac{\partial\overline{e}}{\partial\pi} &= \frac{c\left(\overline{v}+\overline{V}\right)}{\pi\left.\frac{\partial\mathcal{V}_2(e_1)}{\partial e_1}\right|_{e_1=\overline{e}}}.
\end{aligned}$$

By $\left.\frac{\partial\mathcal{V}_2(e_1)}{\partial e_1}\right|_{e_1=\overline{e}}<0$, we conclude $\frac{\partial\overline{e}}{\partial\pi}<0$.

Finally, by $\mathcal{V}_2\left(\overline{e}\right)=(1-c)\left(\frac{2-\pi}{2}\overline{v}+\overline{V}\right)$, note that the preventive-war constraint never binds in equilibrium when

$$\mathcal{V}_2\left(\widehat{e}\right)\geq\frac{1}{2}(1-c)\left(\overline{v}+2\overline{V}\right).$$

In such a case, we have $e^*=\widehat{e}$ for all $\pi\in(0,1)$, and thus e^* is increasing in π. If $\mathcal{V}_2\left(\widehat{e}\right)\big|_{\pi=1}<\frac{1}{2}(1-c)\left(\overline{v}+2\overline{V}\right)$, there exists some $\widehat{\pi}\in(0,1)$ above which $e^*=\overline{e}$ and below which $e^*=\widehat{e}$. Thus, e^* is increasing in π for $\pi\leq\widehat{\pi}$ and decreasing in π for $\pi\geq\widehat{\pi}$. □

Proof of Proposition 5. Consider $t=2$. As in Proposition 1, neither player fights when $D=0$. When $D=1$, by rearranging inequality (4), player i attacks player j if and only if

$$z_j<z_i\frac{(1-2c)\overline{V}-c\overline{v}z_i}{\overline{V}+c\overline{v}z_i}\equiv\underline{z}_j(z_i).$$

Note that $\underline{z}_j(z_i)<z_i$ for any $z_i>0$. Thus, at most one player can have the incentive to attack the other.

Next, consider the decision whether to fight in $t=1$ given investment levels $e=(e_1,e_2)$. Let $u_i(z_i,z_j)$ be the payoff in $t=2$, conditional on $D=1$, when $Z_i=z_i$ and $Z_j=z_j$, excluding the investment cost. Then, we have

$$
\begin{aligned}
u_i(z_i, z_j) \quad &= \quad \overbrace{\mathbb{1}\left\{z_i \geq \underline{z}_i(z_j)\right\} \mathbb{1}\left\{z_j \geq \underline{z}_j(z_i)\right\}\left(z_i \overline{v} + \overline{V}\right)}^{\text{No war}} \\
&\quad + \underbrace{\left(1 - \mathbb{1}\left\{z_i \geq \underline{z}_i(z_j)\right\} \mathbb{1}\left\{z_j \geq \underline{z}_j(z_i)\right\}\right) p_{i,2}(v_2)(1-c)\left((z_i + z_j)\overline{v} + 2\overline{V}\right)}_{\text{War}} \qquad (12)
\end{aligned}
$$

and, consequently,

$$
\mathcal{V}_i^2(e) = (1-\pi)\left(\overline{v} + \overline{V}\right) + \pi \int_0^1 \int_0^1 u_i(z_i, z_j) f_i(z_i|e_i) f_j(z_j|e_j) \mathrm{d}z_i \mathrm{d}z_j. \qquad (13)
$$

Because war in $t = 1$ prevents players' investments from completion and because $\mathbb{E}\left[Z_1|e_1 = 0\right] = \mathbb{E}\left[Z_2|e_2 = 0\right] = 1/2$ by Assumption 5, payoff from preventive war in $t = 1$ is

$$
\frac{1-c}{2}\left[\left(2(1-\pi) + \pi\left(\mathbb{E}\left[Z_1|e_1 = 0\right] + \mathbb{E}\left[Z_2|e_2 = 0\right]\right)\right)\overline{v} + 2\overline{V}\right] = \mathcal{W}^0.
$$

Because investment costs are sunk, player i attacks player j in $t = 1$ if and only if $\mathcal{V}_i^2(e) < \mathcal{W}^0$.

I show that there exists at most one point $\overline{e}_j(e_i)$ at which $\mathcal{V}_i^2\left(e_i, \overline{e}_j(e_i)\right) = \mathcal{W}^0$. Let the peace and war payoffs in $t = 2$ with $D = 1$ in (12) denote $u_i^{\mathtt{P}}(z)$ and $u_i^{\mathtt{W}}(z)$, respectively. It is clear that $u_i^{\mathtt{P}}(z_i, z_j)$ is strictly increasing in z_i and independent of z_j. Also, differentiation yields

$$
\begin{aligned}
\frac{\partial u_i^{\mathtt{W}}(z_i, z_j)}{\partial z_i} \quad &= \quad (1-c)\left(\overline{v} + \frac{2\overline{V}}{(z_i + z_j)^2} z_j\right) > 0 \text{ and} \\
\frac{\partial u_i^{\mathtt{W}}(z_i, z_j)}{\partial z_j} \quad &= \quad -\frac{2(1-c)\overline{V} z_i}{(z_i + z_j)^2} < 0.
\end{aligned}
$$

Now fix player j's proportion of remaining resources after disaster, z_j. Suppose player i's remaining resources increase from z_i' to $z_i'' > z_i'$. Then, the following occurs in $t = 2$ when $D = 1$. First, suppose player j is willing to attack i when $Z = (z_i', z_j)$. Now suppose $Z_i = z_i''$

increases and player j becomes indifferent between fighting and not fighting at $Z = (z_i'', z_j)$. Then, we have

$$\begin{aligned} u_i^{\mathtt{P}}(z_i'', z_j) - u_i^{\mathtt{W}}(z_i'', z_j) &= \left(z_i''\overline{v} + \overline{V}\right) - \frac{z_i''}{z_i'' + z_j}(1-c)\left((z_i'' + z_j)\,\overline{v} + 2\overline{V}\right) \\ &= c\left((z_i'' + z_j)\,\overline{v} + 2\overline{V}\right) \\ &> 0, \end{aligned}$$

which implies that u_i discontinuously increases when player j becomes indifferent between fighting and not fighting at this threshold. Second, suppose that neither player is willing to fight when $D = 1$ under $Z = (z_i', z_j)$. Now suppose $Z_i = z_i''$ and player i is indifferent between fighting and not fighting. This means $z_j = \underline{z}_j(z_i'')$. Then, substituting $\underline{z}_j(z_i'') = z_i'' \frac{(1-2c)\overline{V} - c\overline{v}z_i''}{\overline{V} + c\overline{v}z_i''}$, we obtain

$$u_i^{\mathtt{P}}\left(z_i'', \underline{z}_j(z_i'')\right) - u_i^{\mathtt{W}}\left(z_i'', \underline{z}_j(z_i'')\right) = 0.$$

Hence, u_i is continuously increasing in z_i at this threshold. In the same manner, one can show that u_i is decreasing in z_j.

Thus, by (13) and Assumption 1, $\mathcal{V}_i^2$ is strictly increasing in e_i and strictly decreasing in e_j. Because $\mathcal{W}^0$ does not depend on e_i or e_j, $\mathcal{V}_i^2$ and $\mathcal{W}^0$ cross at most once as e_j increases, holding e_i fixed. Therefore, player i's optimal decision whether to fight in $t = 1$ is $a_{i,1} = \mathbb{1}\{e_j > \overline{e}_j(e_i)\}$. When $\mathcal{V}_i^2(e_i, \overline{e}_j(e_i)) = \mathcal{W}^0$ does not hold, we interpret $\overline{e}_j(e_i) = \infty$.

Next, I show that there cannot be a war in $t = 1$ on the equilibrium path. Suppose to the contrary that player i attacks j in $t = 1$ on the equilibrium path. In such a case, it must be that $e_i = 0$ because a first-period war prevents adaptation from completion and investment costs are sunk. Otherwise, i would have the incentive to deviate to a smaller investment for any $e_i > 0$, and if a first-period war occurs in equilibrium, players' investment must be

$e = (0,0)$. By Assumption 5, we know $\mathcal{V}_1^2(0,0) = \mathcal{V}_2^2(0,0)$. We want to show $\mathcal{V}_i^2(0,0) \geq \mathcal{W}^0$. To this end, let $\mathcal{Z}^{\mathsf{P}} = \{(z_1, z_2) | z_1 \geq \underline{z}_1(z_2), z_2 \geq \underline{z}_2(z_1)\}$ indicate the pairs of Z_1 and Z_2 under which peace remains in $t = 2$. Also, recall that the total amount of resources in society when $D = 1$ is $(Z_1 + Z_2)\overline{v} + 2\overline{V}$ when $Z \in \mathcal{Z}^{\mathsf{P}}$ and $(1-c)\left((Z_1 + Z_2)\overline{v} + 2\overline{V}\right)$ when $Z \notin \mathcal{Z}^{\mathsf{P}}$. Then, because of the symmetry of players' vulnerability under no investment by Assumption 5, we have

$$
\begin{aligned}
\mathcal{V}_i^2(0,0) &= (1-\pi)\left(\overline{v} + \overline{V}\right) + \pi \frac{1}{2} \mathbb{E}_{Z \sim F(\cdot|0,0)} \left[\mathbb{1}\left\{Z \in \mathcal{Z}^{\mathsf{P}}\right\} \left((Z_1 + Z_2)\overline{v} + 2\overline{V}\right) \right. \\
&\quad \left. + \mathbb{1}\left\{Z \notin \mathcal{Z}^{\mathsf{P}}\right\} (1-c) \left((Z_1 + Z_2)\overline{v} + 2\overline{V}\right) \right] \\
&= (1-\pi)\left(\overline{v} + \overline{V}\right) + \pi \frac{1}{2} \left[(1-c)\left((Z_1 + Z_2)\overline{v} + 2\overline{V}\right) \right. \\
&\quad \left. + c\mathbb{E}_{Z \sim F(\cdot|0,0)} \left[\mathbb{1}\left\{Z \in \mathcal{Z}^{\mathsf{P}}\right\} \left((Z_1 + Z_2)\overline{v} + 2\overline{V}\right) \right] \right].
\end{aligned}
$$

By $\mathcal{W}^0 = (1-c)\left(\frac{2-\pi}{2}\overline{v} + \overline{V}\right)$, we obtain

$$
\begin{aligned}
\mathcal{V}_i^2(0,0) - \mathcal{W}^0 &= c(1-\pi)\left(\overline{v} + \overline{V}\right) + \frac{\pi c}{2} \mathbb{E}_{Z \sim F(\cdot|0,0)} \left[\mathbb{1}\{Z \in \mathcal{Z}^{\mathsf{P}}\} \left((Z_1 + Z_2)\overline{v} + 2\overline{V}\right) \right] \\
&> 0.
\end{aligned}
$$

The inequality implies that it is not optimal for player i to attack j under $e = (0,0)$, a contradiction.

Consider the optimal investment level. First, given the other player j's investment e_j, there exists a unique investment level

$$
\widehat{e}_i(e_j) \equiv \arg\max_{e_i} \left[\mathcal{V}_i^2(e_i, e_j) - \kappa_i e_i \right].
$$

One can prove this by showing $\mathcal{V}_i^2(e_i, e_j)$ is strictly concave in e_i. To this end, fix some e_j, and denote $\overline{u}_i(z_i | e_j) = \int_0^1 u_i(z_i, z_j) f_j(z_j | e_j) \mathrm{d}z_j$. Then, by (13) and by integration by parts, we can rewrite player i's continuation value as

$$
\begin{aligned}
\mathcal{V}_i^2(e) &= (1-\pi)\left(\overline{v}+\overline{V}\right)+\pi\int_0^1 \overline{u}_i(z_i|e_j)f_i(z_i|e_i)\mathrm{d}z_i \\
&= (1-\pi)\left(\overline{v}+\overline{V}\right)+\pi\left[\overline{u}_i(1|e_j)-\int_0^1 \frac{\partial\overline{u}_i(z_i|e_j)}{\partial z_i}F_i(z_i|e_i)\mathrm{d}z_i\right].
\end{aligned}
$$

Differentiating with respect to e_i, we obtain

$$
\begin{aligned}
\frac{\partial\mathcal{V}_i^2(e_i,e_j)}{\partial e_i} &= -\pi\int_0^1 \underbrace{\frac{\partial\overline{u}_i(z_i|e_j)}{\partial z_i}}_{>0}\underbrace{\frac{\partial F_i(z_i|e_i)}{\partial e_i}}_{<0 \text{ by Assumption 2}}\mathrm{d}z_i > 0 \text{ and} \\
\frac{\partial^2\mathcal{V}_i^2(e_i,e_j)}{\partial e_i^2} &= -\pi\int_0^1 \frac{\partial\overline{u}_i(z_i|e_j)}{\partial z_i}\underbrace{\frac{\partial^2 F_i(z_i|e_i)}{\partial e_i^2}}_{>0 \text{ by Assumption 2}}\mathrm{d}z_i < 0.
\end{aligned}
$$

Hence, $\mathcal{V}_i^2(e_i,e_j)-\kappa_i e_i$ is strictly concave in e_i. This implies that there is a unique optimum, and denote this optimal investment level by $\widehat{e}_i(e_j)$.

Finally, we specify the equilibrium investment denoted by e_i^2. Peace in $t=1$ requires $\mathcal{V}_1^2(e_1,e_2)\geq\mathcal{W}^0$ and $\mathcal{V}_2^2(e_1,e_2)\geq\mathcal{W}^0$. Let $\underline{e}_i(e_j)$ be the solution to $\mathcal{V}_i^2\left(e_i=\underline{e}_i(e_j),e_j\right)=\mathcal{W}^0$, at which i herself is indifferent between fighting and not fighting in $t=1$. Then, because $\min\left\{\mathcal{V}_1^2(e_1,e_2),\mathcal{V}_2^2(e_1,e_2)\right\}\geq\mathcal{W}^0$ requires $e_i\in[\underline{e}_i(e_j),\overline{e}_i(e_j)]$ for each i given e_j, the optimal investment level is

$$
\max\left\{\underline{e}_i\left(e_j\right),\min\left\{\widehat{e}_i\left(e_j\right),\overline{e}_i\left(e_j\right)\right\}\right\}.
$$

Therefore, the equilibrium investments (e_1^2,e_2^2) solve

$$
\begin{cases}
e_1^2=\max\left\{\underline{e}_1\left(e_2^2\right),\min\left\{\widehat{e}_1\left(e_2^2\right),\overline{e}_1\left(e_2^2\right)\right\}\right\} \\
e_2^2=\max\left\{\underline{e}_2\left(e_1^2\right),\min\left\{\widehat{e}_2\left(e_1^2\right),\overline{e}_2\left(e_1^2\right)\right\}\right\},
\end{cases}
$$

which completes the proof. $\square$

Proof of Proposition 6. Note that the equilibrium probability of war is $\pi\left[\rho_1(e_1,e_2)+\rho_2(e_1,e_2)\right]$. Then, the equilibrium probability of war when $e=(e_1^{\mathfrak{2}},e_2^{\mathfrak{2}})$ is strictly greater than the probability when $e=(0,0)$ if and only if

$$\begin{aligned}\pi\left[\rho_1\left(e_1^{\mathfrak{2}},e_2^{\mathfrak{2}}\right)+\rho_2\left(e_1^{\mathfrak{2}},e_2^{\mathfrak{2}}\right)\right] &> \pi\left[\rho_1\left(0,0\right)+\rho_2\left(0,0\right)\right]\\ -\rho_1\left(0,0\right)-\rho_2\left(0,0\right) &> -\rho_1\left(e_1^{\mathfrak{2}},e_2^{\mathfrak{2}}\right)-\rho_2\left(e_1^{\mathfrak{2}},e_2^{\mathfrak{2}}\right).\end{aligned}$$

Adding $\rho_1\left(e_1^{\mathfrak{2}},0\right)+\rho_2\left(0,e_2^{\mathfrak{2}}\right)$ to both sides yields $\tau\left(e^{\mathfrak{2}}\right)>\delta\left(e^{\mathfrak{2}}\right)$. $\square$

Proof of Proposition 7. Fix any investment profile $e=(e_1,e_2)$ with $e_1,e_2\geq 0$. By MLRP (2),

$$\frac{\partial}{\partial z}\frac{F_i(z\mid e_i)}{F_i(z\mid 0)} = \frac{\displaystyle\int_0^z \left[f_i(z\mid e_i)f_i(z'\mid 0)-f_i(z\mid 0)f_i(z'\mid e_i)\right]\mathrm{d}z'}{F_i(z\mid 0)^2}\geq 0.$$

Thus, $F_i(z\mid e_i)/F_i(z\mid 0)$ is nondecreasing in z. Because $0<\underline{z}_j(z)<z$ for $z\in(0,1]$, this implies

$$F_j(\underline{z}_j(z)\mid e_j)\leq\frac{F_j(\underline{z}_j(z)\mid 0)}{F_j(z\mid 0)}F_j(z\mid e_j).$$

By Assumption 5, $F_1(z\mid 0)=F_2(z\mid 0)$. The attack thresholds also satisfy $\underline{z}_1(z)=\underline{z}_2(z)$. Using the definition of ρ_i, we therefore obtain

$$\begin{aligned}&\rho_1(e_1,e_2)+\rho_2(e_1,e_2)\\ =&\int_0^1 F_2(\underline{z}_2(z)\mid e_2)f_1(z\mid e_1)\,\mathrm{d}z+\int_0^1 F_1(\underline{z}_1(z)\mid e_1)f_2(z\mid e_2)\,\mathrm{d}z\\ \leq&\int_0^1\frac{F_1(\underline{z}_1(z)\mid 0)}{F_1(z\mid 0)}\frac{\partial}{\partial z}\left[F_1(z\mid e_1)F_2(z\mid e_2)\right]\,\mathrm{d}z.\end{aligned}$$

Condition (3) implies

$$\frac{\partial}{\partial z}\frac{F_1(\underline{z}_1(z) \mid 0)}{F_1(z \mid 0)} \leq 0.$$

Moreover, Assumptions 1 and 5 give

$$F_1(z \mid e_1)F_2(z \mid e_2) \leq F_1(z \mid 0)^2.$$

Integrating by parts and applying these inequalities yields

$$\begin{aligned}
& \int_0^1 \frac{F_1(\underline{z}_1(z) \mid 0)}{F_1(z \mid 0)} \frac{\partial}{\partial z}\left[F_1(z \mid e_1)F_2(z \mid e_2)\right] \mathrm{d}z \\
= \;& F_1(\underline{z}_1(1) \mid 0) - \int_0^1 F_1(z \mid e_1)F_2(z \mid e_2)\frac{\partial}{\partial z}\left[\frac{F_1(\underline{z}_1(z) \mid 0)}{F_1(z \mid 0)}\right] \mathrm{d}z \\
\leq \;& F_1(\underline{z}_1(1) \mid 0) - \int_0^1 F_1(z \mid 0)^2\frac{\partial}{\partial z}\left[\frac{F_1(\underline{z}_1(z) \mid 0)}{F_1(z \mid 0)}\right] \mathrm{d}z \\
= \;& 2\int_0^1 F_1(\underline{z}_1(z) \mid 0) f_1(z \mid 0)\,\mathrm{d}z \\
= \;& \rho_1(0,0) + \rho_2(0,0).
\end{aligned}$$

The boundary terms at $z = 0$ vanish because $0 \leq F_1(\underline{z}_1(z) \mid 0)/F_1(z \mid 0) \leq 1$ and the products of CDFs converge to zero as $z \downarrow 0$. The final equality follows from baseline symmetry and the common attack-threshold formula.

By Proposition 5, no war occurs in $t = 1$ on the equilibrium path. Applying the preceding inequality at $e = (e_1^2, e_2^2)$ and multiplying by π therefore gives

$$\pi\left[\rho_1(e_1^2, e_2^2) + \rho_2(e_1^2, e_2^2)\right] \leq \pi\left[\rho_1(0,0) + \rho_2(0,0)\right].$$

This is the desired comparison of the ex ante probabilities of war. $\square$